An efficient and simple class of functions to model arrival curve of packetised flows

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Outline

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What is Network Calculus ?

- A theory designed to compute *guaranteed bounds* on delays.
- With a strong mathematical background: (min,+) algebra
  - Basic object: non-decreasing, non-negative functions

$$\mathcal{F} = \{ f : \mathbb{R}_+ \to \mathbb{R}_+ \mid x < y \implies f(x) \leq f(y) \}$$

  - Three basic operations: the convolution $*$, deconvolution $\oslash$, the sub-additive closure f^*.

$$(f * g)(t) = \inf_{0 \leq u \leq t} (f(t-u) + g(u)) \tag{1}$$

$$(f \oslash g)(t) = \sup_{0 \leq u} (f(t+u) - g(u)) \tag{2}$$

$$f^* = \delta_0 \wedge f \wedge (f * f) \wedge (f * f * f) \wedge \cdots \tag{3}$$

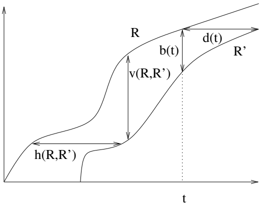
Network calculus overview

Two basic objects:

- *Flow:*
  - modelling: $R \in \mathcal{F} = \{\mathbb{R}^+ \to \mathbb{R}^+, \text{non-decreasing}\}$
  - semantics: $R(t)$, cumulative amount of data up to t
- *Server:*
  - modelling: $S \in \mathcal{F} \times \mathcal{F}$: $R \xrightarrow{S} R' \implies R' \leq R$
  - semantics: relation between some input and some output, no loss, output comes after input ($R'(t) \leq R(t)$)
- *delay:*

$$d(R, S) \quad \leq \quad \max_{R \xrightarrow{S} R'} h(R, R')$$

$h(R, R')$: horizontal deviation



Contract modelling

- Flow contract: arrival curve α

$$R \prec \alpha \iff \forall t, \Delta \in \mathbb{R}^{+} R(t+\Delta) - R(t) \leq \alpha(\Delta)$$
$$\iff R \leq R * \alpha$$

- Server contract: service curve
  - simple service of curve β
    $R \xrightarrow{S} R' \iff R' \geq R * \beta$
  - strict service of curve β
    for all backlogged period $[t, t+\Delta[$
    $(i.e. \forall x \in [t, t+\Delta[$: $R'(x) < R(x))$:
    $R'(t+\Delta) - R'(t) \geq \beta(\Delta)$
- Results: $R \xrightarrow{S} R'$, $R \prec \alpha$, S has service curve β:

$$R' \prec \alpha \oslash \beta$$
$$d(R, S) \leq h(\alpha, \beta)$$

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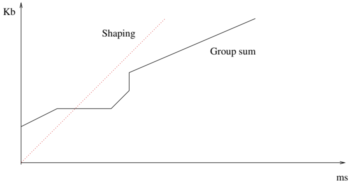
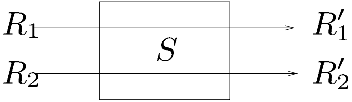
Shaping on links

A link is shared by a set of flows: what is the throughput of this set ?

- Principle: whatever the applicative throughput is, is it limited by the links capacity
  Also known has:
  - Serialisation: the frames of the different flows can not be sent at the same time
  - Grouping: computes per-group throughput, not per-flow
- Interest: considering long term rate ρ and instantaneous burst b
  - applicative flows: small ρ, big b
  - link: big ρ, null b
- Impact: up to 40% in industrial system

Shaping and network calculus





Let S be a server, with *shaping curve* σ, then, the output is constrained by σ.
If the output is constrained by α', it is by $\alpha' \wedge \sigma$.

$$R \xrightarrow{S} R' \implies R' \prec \sigma$$

$$R \prec \alpha, S \trianglerighteq \beta \implies R' \prec \sigma \wedge (\alpha \oslash \beta)$$

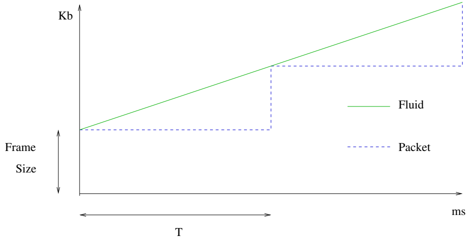
Modelling a packetized flow

Common example: sporadic flow

- inter emission "period": T
- frame size (fixed or max): b

Two modelling:

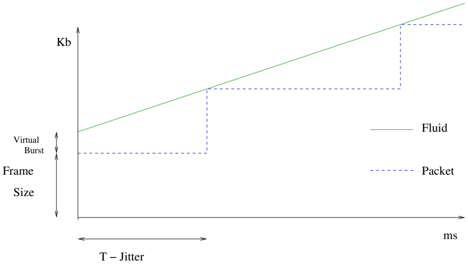
- fluid ("token bucket"): affine function, continuous
- packetized: stair-case functions, discontinuous



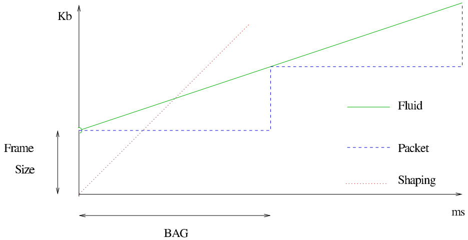
Fluid modelling: the virtual burst problem

Jitter "shifts" the arrival curve:

- if jitter $<$ period: instantaneous burst unchanged
- in fluid modelling: creation of virtual burst $\Longrightarrow$ increase bounds



Putting all together



- fluid + shaping: concave piecewise linear function (CPL)
  Efficient min, max, sum
  Implementation in floating points
- stair-case modelling: general class (UPP)
  Complex min, max, sum
  Implementation in exact rationals ($\mathbb{Q}$)

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Getting the better of each class

Classes strengths/weaknesses:

- jitter effect: stair-case class
- summing ("grouping"): CPL class
- shaping: CPL class

Idea:

- keeping stair-case for individual flow constraint
- converting into CPL when summing and shaping

From stair-case to CPL

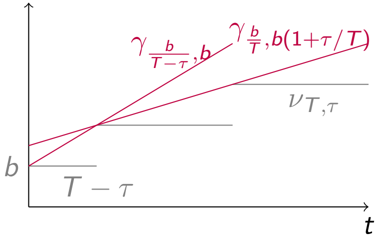

Figure: CPL overapproximation of a stair-case function

$$cpl(b\nu_{T,\tau}) = \gamma_{\frac{b}{nT-\tau},nb} \wedge \gamma_{\frac{b}{T},b(1+\tau/T)} \quad (4)$$

Algorithm adaptation

Function $GroupAC(S_j, \mathcal{F})$;
$\texttt{Assert}(\mathcal{F} \subset S_j{}^\bullet)$;
begin
  if $S_j = S_0$ **then**
    return $\sum_{F_i^k \in \mathcal{F}} \alpha_i^k$; // $\mathcal{F} \subset S_j{}^\bullet \implies k = 0$
  else if S_j *is an aggregate server* **then**
    if $d_j = null$ **then**
      $\alpha^{S_j} \leftarrow \sum_{S_p \in {}^{\bullet\bullet}S_j} \texttt{GroupAC}(S_p, S_p{}^\bullet \cap {}^\bullet S_j)$;
      $d_j = h(\alpha^{S_j}, \beta_j)$;
      foreach $F_i^k \in {}^\bullet S_j$ **do**
        $\alpha_i^{k+1} \leftarrow (\alpha_i^k \oslash \delta_{d_j})^*$;
    return $\left(\sigma_j \wedge \sum_{F_i^k \in \mathcal{F}} \alpha_i^k\right)^*$

Adaptation: replace $\sum_{F_i^k \in \mathcal{F}} \alpha_i^k$ by $\sum_{F_i^k \in \mathcal{F}} cpl(\alpha_i^k)$

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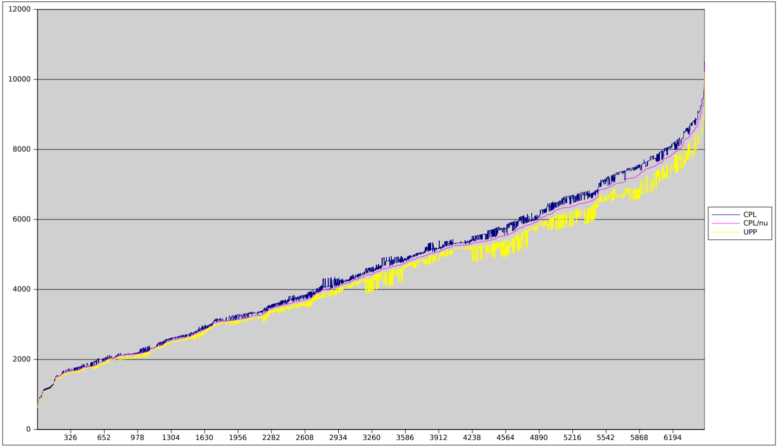
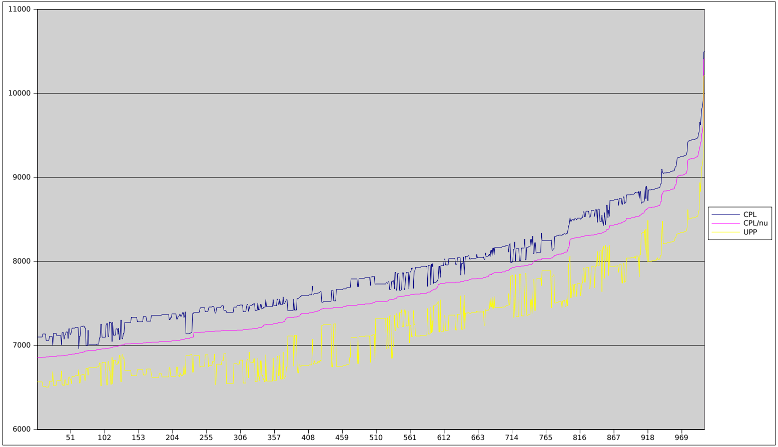
Testbed configuration

- industrial (Thales) configuration
- 104 nodes
- 8 switches
- 974 multicast flows
- 6501 end-to-end bounds

Comparing methods



Zoom on worst delays



Pessimism evaluation

- Method comparison: based on upper bound (UB_m)
- Best comparison: based on pessism

$$\text{pess}_m = UB_m - WCTT$$

- Worst case unknown ($WCTT$)
- Delay lower bound: trajectorial based approach (LB)

$$LB \leq WCTT \leq UB_m$$
$$\text{pess}_m \leq UB_m - LB$$

Uppers and lower bounds

An efficient and simple class

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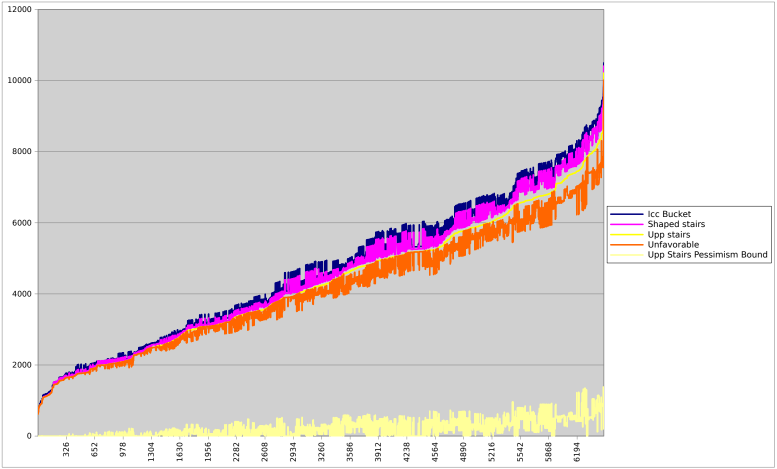
Network calculus

Shaping, packetization and computation time

Swaping between function classes

Experiment

Conclusion



Pessimism bounding, per method

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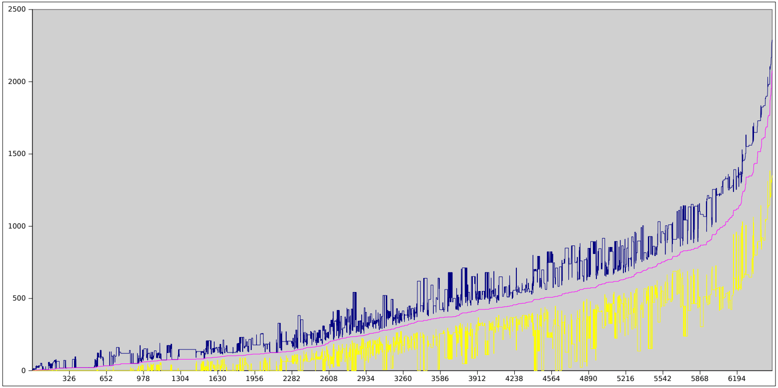

Experiment values

Method	CPL (float)	CPL$/b.\nu_{T,\tau}$ (float)	UPP (rat)
Computation time	0.9 s	1.1 s	7.2 s
Min gain	-	0%	0.15%
Max gain	-	7.8%	15.2%
Av. gain	-	2.49%	5.92%
Min gain on 1000 biggest	-	0.8%	2.0%
Max gain on 1000 biggest	-	4.4%	11.9%
Av. gain on 1000 biggest	-	2.9%	8.3%

Gain correlation: 0.785

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Conclusion

- Two critical aspects: shaping and packetization
- Two existing methods:
  - fluid: bad packetization, quick computation
  - stair-case: good packetization, longer computation
- Contribution: trade-off tightness/computation time
- Don't use CPL, use CPL/$b.\nu_{T,\tau}$
  - simple to implement
  - low computation time over-head
  - significant bound improvement
- Perspective: use in optimisation loop
  - quick computation in first iterations
  - longer computation to finalise